

Hard exclusive electroproduction of two pions and their resonances ¹

M.V. Polyakov

Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188350, Russia
 and
 Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

Abstract

We study the hard exclusive production of two pions in the virtual photon fragmentation region with various invariant masses including the resonance region. The amplitude is expressed in terms of two-pion light cone distribution amplitudes (2π DA's). We derive dispersion relations for these amplitudes, which enables us to fix them completely in terms of $\pi\pi$ scattering phases and a few low-energy subtraction constants determined by the effective chiral lagrangian. Quantitative estimates of the resonance as well $\pi\pi$ background DA's at low normalization point are made. We also prove certain new soft pion theorem relating two-pion DA's to the one-pion DA's. Crossing relations between 2π DA's and parton distributions in a pion are discussed.

We demonstrate that by studying the shape of the $\pi\pi$ mass spectra (not absolute cross section!) in a diffractive electroproduction one can extract the deviation of the meson (π, ρ , etc.) wave functions from their asymptotic form $6z(1-z)$ and hence to get important information about the structure of mesons. We suggest (alternative to Söding's) parametrization of $\pi\pi$ spectra which is suitable for large photon virtuality.

The aim of this talk is to outline the approach to two-pion hard electroproduction developed in [1]. Here we summarize the main results, details can be found in [1].

Hard exclusive electroproduction of mesons is a new kind of hard processes calculable in QCD. In [2, 3] was shown that a number of exclusive processes of the type:

$$\gamma_L^*(q) + T(p) \rightarrow F(q') + T'(p'), \quad (1)$$

at large Q^2 with $t = (p - p')^2$, $x_{Bj} = Q^2/2(pq)$ and $q'^2 = M_F^2$ fixed, are amenable to QCD description. The factorization theorem [2] asserts that the amplitude of the process (1) has the following form:

$$\sum_{i,j} \int_0^1 dz \int dx_1 f_{i/T}^{T'}(x_1, x_1 - x_{Bj}; t, \mu) H_{ij}(Q^2 x_1/x_{Bj}, Q^2, z, \mu) \Phi_j^F(z, \mu) \\ + \text{power-suppressed corrections}, \quad (2)$$

where $f_{i/T}^{T'}$ is $T \rightarrow T'$ skewed parton distribution (for review and references see [4]), $\Phi_j^F(z, \mu)$ is the distribution amplitude of the hadronic state F (not necessarily one particle state), and H_{ij} is a hard part computable in pQCD as series in $\alpha_s(Q^2)$. Here we shall study general properties of the distribution amplitudes $\Phi_j^F(z, \mu)$ when the final hadronic state F is a two pion state ($F = \pi\pi$).

The 2π DA's were introduced recently in [5] in the context of QCD description of the process $\gamma^*\gamma \rightarrow 2\pi$. In ref. [6, 1] it was suggested that the usages of 2π DA's to describe a hard electroproduction of two pions leads to universal picture of resonant and non-resonant two pion production. Below we listed main properties of 2π DA's and shortly discuss their applications to hard diffractive production of two pions off nucleon.

¹Talk given at conference "Particle and Nuclear Physics with CEBAF at Jefferson Lab", Dubrovnik, 3-10 November 1998

• **Definition:**

$$\Phi^{ab}(z, \zeta, m_{\pi\pi}^2) = \frac{1}{4\pi} \int dx^- e^{-\frac{i}{2}zP^+x^-} {}_{out}\langle \pi^a(p_1) \pi^b(p_2) | \bar{\psi}(x) \hat{n} T \psi(0) | 0 \rangle \Big|_{x^+=x_\perp=0}, \quad (3)$$

Here, n is a light-like vector ($n^2 = 0$), which we take as $n_\mu = (1, 0, 0, 1)$. For any vector, V , the “plus” light-cone coordinate is defined as $V^+ \equiv (n \cdot V) = V^0 + V^3$; the “minus” component as $V^- = V^0 - V^3$. The outgoing pions have momenta p_1, p_2 , and $P \equiv p_1 + p_2$ is the total momentum of the final state. Finally, T is a flavor matrix ($T = 1$ for the isosinglet, $T = \tau^3/2$ for the isovector 2π DA). The generalized distribution amplitudes, eq. (3), depend on the following kinematical variables: the quark momentum fraction with respect to the total momentum of the two-pion state, z ; the variable $\zeta \equiv p_1^+/P^+$ characterizing the distribution of longitudinal momentum between the two pions, and the invariant mass, $m_{\pi\pi}^2 = P^2$.

• **The isospin decomposition:**

$$\Phi^{ab} = \delta^{ab} \text{tr}(T) \Phi^{I=0} + \frac{1}{2} \text{tr}([\tau^a, \tau^b] T) \Phi^{I=1}. \quad (4)$$

• **Symmetries and normalization:** From the C-parity one can easily derive the following symmetry properties of the isoscalar ($I = 0$) and isovector ($I = 1$) parts of 2π DA’s eq. (3):

$$\begin{aligned} \Phi^{I=0}(z, \zeta, m_{\pi\pi}^2) &= -\Phi^{I=0}(1-z, \zeta, m_{\pi\pi}^2) = \Phi^{I=0}(z, 1-\zeta, m_{\pi\pi}^2), \\ \Phi^{I=1}(z, \zeta, m_{\pi\pi}^2) &= \Phi^{I=1}(1-z, \zeta, m_{\pi\pi}^2) = -\Phi^{I=1}(z, 1-\zeta, m_{\pi\pi}^2). \end{aligned} \quad (5)$$

The isospin one parts of 2π DA’s eq. (3) is normalized as follows:

$$\int_0^1 dz \Phi^{I=1}(z, \zeta, m_{\pi\pi}^2) = (2\zeta - 1) F_\pi^{\text{e.m.}}(m_{\pi\pi}^2), \quad (6)$$

where $F_\pi^{\text{e.m.}}(m_{\pi\pi}^2)$ is the pion e.m. form factor in the time-like region ($F_\pi^{\text{e.m.}}(0) = 1$).

For the isoscalar 2π DA $\Phi^{I=0}$ we have the following normalization condition:

$$\int_0^1 dz (2z - 1) \Phi^{I=0}(z, \zeta, m_{\pi\pi}^2) = -2 M_2^\pi \zeta (1 - \zeta) F_\pi^{\text{EMT}}(m_{\pi\pi}^2),$$

where M_2^π is a momentum fraction carried by quarks in a pion, $F_\pi^{\text{EMT}}(m_{\pi\pi}^2)$ is a pion form factor of quark part of energy momentum tensor normalized by $F_\pi^{\text{EMT}}(0) = 1$. In ref. [1] this form factor was estimated in the instanton model of QCD vacuum at low two-pion invariant mass, the result:

$$F_\pi^{\text{EMT}}(m_{\pi\pi}^2) = 1 + \frac{N_c m_{\pi\pi}^2}{48\pi^2 f_\pi^2} + \dots,$$

where $f_\pi = 93$ MeV is a pion decay constant.

• **Double decomposition in conformal and partial waves :** Let us decompose 2π DA’s in eigenfunctions of the ERBL [8] evolution equation (Gegenbauer polynomials $C_n^{3/2}(2z - 1)$) and in partial waves of produced pions (Gegenbauer polynomials $C_l^{1/2}(2\zeta - 1)$). Generically the decomposition (for both isoscalar and isovector DA’s) has the form:

$$\Phi(z, \zeta, m_{\pi\pi}^2) = 6z(1-z) \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(m_{\pi\pi}^2) C_n^{3/2}(2z-1) C_l^{1/2}(2\zeta-1). \quad (7)$$

Using symmetry properties Eq. (5) we see that the index n goes over even (odd) and index l goes over odd (even) numbers for isovector (isoscalar) 2π DA’s. Normalization conditions Eq. (6) correspond to $B_{01}^{I=1}(m_{\pi\pi}^2) = F_\pi^{\text{e.m.}}(m_{\pi\pi}^2)$.

• **Relations of 2π DA's to quark distribution in a pion** : By crossing symmetry the 2π DA's are related to so-called skewed parton distributions (see review [4]). The latter are defined as:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda\tau} \langle \pi^a(p') | T \{ \bar{\psi}_{f'}(-\lambda n/2) \hat{n} \psi_f(\lambda n/2) \} | \pi^b(p) \rangle = \delta^{ab} \delta_{ff'} H^{I=0}(\tau, \xi, t) + i \varepsilon^{abc} \tau_{ff'}^c H^{I=1}(\tau, \xi, t) , \quad (8)$$

where ξ is a skewedness parameter defined as: $\xi = -(n \cdot (p' - p)) / (n \cdot (p' + p))$, $t = (p' - p)^2$ and light-cone vector n^μ normalized by $(n \cdot (p' + p)) = 2$. By crossing symmetry we can easily express the moments of skewed distributions to coefficients B_{nl} in the expansion (7) (detailed discussion see in [15]):

$$\int_{-1}^1 d\tau \tau^{N-1} H^I(\tau, \xi, t) = \sum_{n=0}^{N-1} \sum_{l=0}^{n+1} B_{nl}^I(t) \xi^N C_l^{1/2} \left(\frac{1}{\xi} \right) \int_{-1}^1 dx \frac{3}{4} [1 - x^2] x^{N-1} C_n^{3/2}(x) . \quad (9)$$

If we take the forward limit in this formula, we obtain the relations between moments of quark distributions in a pion and coefficients B_{nl} :

$$\begin{aligned} M_N^{(\pi)} &\equiv \int_0^1 x^{N-1} (q_\pi(x) - \bar{q}_\pi(x)) = B_{N-1,N}^{I=1}(0) A_N \quad \text{for odd } N , \\ M_N^{(\pi)} &\equiv 2 \int_0^1 x^{N-1} (q_\pi(x) + \bar{q}_\pi(x)) = B_{N-1,N}^{I=0}(0) A_N \quad \text{for even } N , \end{aligned} \quad (10)$$

where A_N are numerical coefficients (*e.g.* $A_1 = 1, A_2 = 9/5, A_3 = 6/7, A_4 = 5/3$, etc.) and $q_\pi(x) = u^{\pi^+}(x)$. For example, from eq. (10) we obtain that $B_{01}^{I=1}(0) = M_1^{(\pi)} = 1$ what corresponds to normalization condition (6). Also it is easy to see that $B_{12}^{I=0}(0) = 5/9 M_2^{(\pi)}$ corresponds to normalization (7)².

• **Evolution** : The Gegenbauer polynomials $C_n^{3/2}(2z - 1)$ are eigenfunctions of the ERBL [8] evolution equation and hence the coefficients B_{nl} are renormalized multiplicatively (for even n and odd l):

$$B_{nl}(m_{\pi\pi}^2; \mu) = B_{nl}(m_{\pi\pi}^2; \mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_n / (2\beta_0)} , \quad (11)$$

where $\beta_0 = 11 - 2/3 N_f$ and the one loop anomalous dimensions are [9]:

$$\gamma_n = \frac{8}{3} \left(1 - \frac{2}{(1+n)(2+n)} + 4 \sum_{k=2}^{n+1} \frac{1}{k} \right) .$$

From the decomposition Eq. (7) and Eq. (11) we can make a simple prediction for the ratio of the hard P - and F - waves production amplitudes of pions in the reaction $\gamma^* p \rightarrow 2\pi p$ at asymptotically large virtuality of the incident photon $Q^2 \rightarrow \infty$ and fixed $m_{\pi\pi}^2$:

$$\frac{F\text{-wave amplitude}}{P\text{-wave amplitude}} \sim \frac{1}{\log(Q^2)^{50/(99-6N_f)}} , \quad (12)$$

or generically for the $2k + 1$ wave:

$$\frac{(2k+1)\text{-wave amplitude}}{P\text{-wave amplitude}} \sim \frac{1}{\log(Q^2)^{\gamma_{2k}/(2\beta_0)}} . \quad (13)$$

• **Soft pion theorems for two-pion distribution amplitudes** relate 2π DA's to distribution amplitude of one pion:

²To see this one has to use additionally soft pion theorem (15) for isoscalar 2π DA

$$\Phi^{I=1}(z, \zeta = 1, m_{\pi\pi}^2 = 0) = -\Phi^{I=1}(z, \zeta = 0, m_{\pi\pi}^2 = 0) = \varphi_\pi(z), \quad (14)$$

where $\varphi_\pi(z)$ is a one pion DA. The analogous theorem for an isoscalar part of the 2π DA's has the form:

$$\Phi^{I=0}(z, \zeta = 1, m_{\pi\pi}^2 = 0) = \Phi^{I=0}(z, \zeta = 0, m_{\pi\pi}^2 = 0) = 0. \quad (15)$$

The soft pion theorem Eq. (14) allows us to relate the coefficients $B_{nl}^{I=1}(m_{\pi\pi}^2)$ (see Eq. (7)) and the coefficients of expansion of the pion DA in Gegenbauer polynomials

$$\varphi_\pi(z) = 6z(1-z) \left(1 + \sum_{n=\text{even}} a_n^{(\pi)} C_n^{3/2}(2z-1) \right). \quad (16)$$

The relation has the form:

$$a_n^{(\pi)} = \sum_{l=1}^{n+1} B_{nl}^{I=1}(0). \quad (17)$$

• **Dispersion relations and their solution for 2π DA's:** The 2π DA's are generically complex functions due to the strong interaction of the produced pions. Above the two-pion threshold $m_{\pi\pi}^2 = 4m_\pi^2$ the 2π DA's develop the imaginary part corresponding to the contribution of on-shell intermediate states (2π , 4π , etc.). In the region $m_{\pi\pi}^2 < 16m_\pi^2$ the imaginary part is related to the pion-pion scattering amplitude by Watson theorem [10]. This relation can be written in the following form (see [1]):

$$\text{Im } B_{nl}^I(m_{\pi\pi}^2) = \sin[\delta_l^I(m_{\pi\pi}^2)] e^{i\delta_l^I(m_{\pi\pi}^2)} B_{nl}^I(m_{\pi\pi}^2)^* = \tan[\delta_l^I(m_{\pi\pi}^2)] \text{Re } B_{nl}^I(m_{\pi\pi}^2). \quad (18)$$

Using Eq. (18) one can write an N -subtracted dispersion relation for the $B_{nl}^I(m_{\pi\pi}^2)$

$$B_{nl}^I(m_{\pi\pi}^2) = \sum_{k=0}^{N-1} \frac{m_{\pi\pi}^{2k}}{k!} \frac{d^k}{dm_{\pi\pi}^{2k}} B_{nl}^I(0) + \frac{m_{\pi\pi}^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\tan \delta_l^I(s) \text{Re } B_{nl}^I(s)}{s^N (s - m_{\pi\pi}^2 - i0)}. \quad (19)$$

Solution of such type dispersion relation was found long ago in [11], the solution has the exponential form:

$$B_{nl}^I(m_{\pi\pi}^2) = B_{nl}^I(0) \exp \left\{ \sum_{k=1}^{N-1} \frac{m_{\pi\pi}^{2k}}{k!} \frac{d^k}{dm_{\pi\pi}^{2k}} \log B_{nl}^I(0) + \frac{m_{\pi\pi}^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_l^I(s)}{s^N (s - m_{\pi\pi}^2 - i0)} \right\}. \quad (20)$$

A great advantage of the solution Eq. (20) is that it gives the 2π DA's in a wide range of energies in terms of known $\pi\pi$ phase shifts and a few subtraction constants (usually two is sufficient). The key observation is that these constants (non-perturbative input) are related to the *low-energy* behaviour of the 2π DA's at $m_{\pi\pi}^2 \rightarrow 0$. In the low energy region they can be computed using the effective chiral lagrangian.

• **Amplitudes of $\pi\pi$ resonances production:** We give here an example how ρ meson distribution amplitude can be expressed in terms of 2π DA. General formula for DA's of resonances with any spin can be found in [1]. In [1] we obtained the following expression for the coefficients of the expansion of ρ meson DA in Gegenbauer polynomials

$$\varphi_\rho(z) = 6z(1-z) \left(1 + \sum_{n=\text{even}} a_n^{(\rho)} C_n^{3/2}(2z-1) \right),$$

in terms of only subtraction constants entering Eq. (20)

$$a_n^{(\rho)} = B_{n1}^{I=1}(0) \exp \left(\sum_{k=1}^{N-1} c_k^{(n1)} m_\rho^{2k} \right), \quad (21)$$

where the subtraction constants $c_k^{(nl)}$ can be expressed in terms of $B_{nl}^{I=1}(m_{\pi\pi}^2)$ at low energies

$$c_k^{(nl)} = \frac{1}{k!} \frac{d^k}{dm_{\pi\pi}^{2k}} [\log B_{nl}(m_{\pi\pi}^2) - \log B_{l-1,l}(m_{\pi\pi}^2)] \Big|_{m_{\pi\pi}^2=0}. \quad (22)$$

The normalization constants f_ρ can be computed as:

$$f_\rho = \frac{\sqrt{2}\Gamma_\rho \text{Im} F_\pi^{\text{e.m.}}(m_\rho^2)}{g_{\rho\pi\pi}}. \quad (23)$$

Here we quote the result for the chirally even ρ meson DA extracted from 2π DA using Eqs. (21,23) and values of low-energy subtraction constants calculated using effective chiral lagrangian (see details and results of these calculations in refs. [6, 1]) :

$$\varphi_\rho(z) = 6z(1-z)[1 - 0.14 C_2^{3/2}(2z-1) - 0.01 C_4^{3/2}(2z-1) + \dots], \quad (24)$$

with normalization constant $f_\rho = 190$ MeV according to Eq. (23) in a good agreement with experimental value $f_\rho = 195 \pm 7$ MeV [14]. The ρ meson DA's were the subject of the QCD sum rules calculations [12, 13], our result Eq. (24) is in a qualitative disagreement with the results of QCD sum rule calculations, the sign of a_2 obtained here is opposite to the QCD sum rules predictions $a_2^{(\rho)} = 0.18 \pm 0.1$ [12] and $a_2^{(\rho)} = 0.08 \pm 0.02$, $a_4^{(\rho)} = -0.08 \pm 0.03$ [13] (these results refer to normalization point $\mu = 1$ GeV). Let us note that actually our sign of $a_2^{(\rho)}$ follows from: 1) soft pion theorem (14) 2) the relation of $B_{N-1,N}$ to moments of quark distribution in a pion (10) 3) relation $a_2^{(\rho)} = B_{21}^{I=1}(0) \exp(c_1^{(21)} m_\rho^2)$ (see eq.(21)) and 4) the fact that $a_2^{(\pi)} \approx 0$. Combining all this we can state that:

$$\text{sign}(a_2^{(\rho)}) = \text{sign}(B_{21}^{I=1}(0)) = \text{sign}(a_2^{(\pi)} - B_{23}^{I=1}(0)) = \text{sign}(a_2^{(\pi)} - \frac{7}{6} M_3^{(\pi)}) = -\text{sign}(M_3^{(\pi)}). \quad (25)$$

Here in the last equality we assumed that $a_2^{(\pi)} < \frac{7}{6} M_3^{(\pi)}$ what is satisfied in the model and seems phenomenologically.

• QCD parametrization of two pion spectrum in diffractive two pion production: The dependence of the two-pion hard production amplitude on the $m_{\pi\pi}$ factorizes into the factor:

$$A \propto \int_0^1 \frac{dz}{z(1-z)} \Phi^{I=1}(z, \zeta, m_{\pi\pi}^2; \bar{Q}^2), \quad (26)$$

for the two pions in the isovector state, and

$$A \propto \int_0^1 dz \frac{(2z-1)}{z(1-z)} \Phi^{I=0}(z, \zeta, m_{\pi\pi}^2; \bar{Q}^2), \quad (27)$$

for the pions in the isoscalar state (*e.g.* for the $\pi^0\pi^0$ production). In the above equations we showed also the dependence of the 2π DA's on the scale of the process \bar{Q}^2 , which is governed by the ERBL evolution equation [8].

For the hard exclusive reactions off nucleon at small x_{Bj} (see *e.g.* recent measurements [7]) the production of two pions in the isoscalar channel is strongly suppressed relative to the isovector channel because the former is mediated by C -parity odd exchange. At asymptotically large Q^2 one can use the asymptotic form of isovector 2π DA:

$$\lim_{Q^2 \rightarrow \infty} \Phi^{I=1}(z, \zeta, m_{\pi\pi}^2; \bar{Q}^2) = 6F_\pi^{\text{e.m.}}(m_{\pi\pi}^2) z(1-z)(2\zeta-1), \quad (28)$$

where $F_\pi^{\text{e.m.}}(m_{\pi\pi}^2)$ is the pion e.m. form factor in the time-like region. Therefore the shape of $\pi^+\pi^-$ mass spectrum in the hard electroproduction process at small x_{Bj} and asymptotically large Q^2 should be determined completely by the pion e.m. form factor in time-like region:

$$\lim_{Q^2 \rightarrow \infty} A \propto e^{i\delta_1^1(m_{\pi\pi}^2)} |F_\pi^{\text{e.m.}}(m_{\pi\pi}^2)| (2\zeta - 1). \quad (29)$$

Deviation of the $\pi^+\pi^-$ mass spectrum from its asymptotic form Eq. (29) (“skewing”) can be parametrized at small x_{Bj} and large \bar{Q}^2 in the form:

$$\begin{aligned} A \propto e^{i\delta_1^1(m_{\pi\pi}^2)} |F_\pi^{\text{e.m.}}(m_{\pi\pi}^2)| & \left[1 + B_{21}(0; \mu_0) \exp\{c_1^{(21)} m_{\pi\pi}^2\} \left(\frac{\alpha_s(\bar{Q}^2)}{\alpha_s(\mu_0)} \right)^{50/(99-6N_f)} \right] (2\zeta - 1) + \\ & e^{i\delta_3^1(m_{\pi\pi}^2)} B_{23}(0; \mu_0) \exp\{b_{23} m_{\pi\pi}^2 + R_3^1(m_{\pi\pi}^2)\} \left(\frac{\alpha_s(\bar{Q}^2)}{\alpha_s(\mu_0)} \right)^{50/(99-6N_f)} C_3^{1/2} (2\zeta - 1) \\ & + \text{higher powers of } 1/\log(\bar{Q}^2), \end{aligned} \quad (30)$$

here

$$R_l^I(m_{\pi\pi}^2) = \text{Re} \frac{m_{\pi\pi}^4}{\pi} \int_{4m_\pi^2}^\infty ds \frac{\delta_l^I(s)}{s^2(s - m_{\pi\pi}^2 - i0)} \quad \text{and} \quad b_{nl} = \frac{d}{dm_{\pi\pi}^2} \log B_{nl}(m_{\pi\pi}^2) \Big|_{m_{\pi\pi}^2=0}. \quad (31)$$

We see that the deviation of the $\pi\pi$ invariant mass spectrum from its asymptotic form Eq. (29) in this approximation is characterized by a few low-energy constants ($B_{21}(0)$, $B_{23}(0)$, $c_1^{(21)}$, b_{23}), other quantities—the pion e.m. form factor and the $\pi\pi$ phase shifts—are known from low-energy experiments. In principle, using the parametrization (31) one can extract the values of these low-energy parameters from the shape of $\pi\pi$ spectrum (not absolute cross section!) in diffractive production experiments. Knowing them one can obtain the deviation of the π meson DA (see Eq. (17))

$$a_2^{(\pi)} = B_{21}(0) + B_{23}(0),$$

and the ρ meson DA (see Eq. (21))

$$a_2^{(\rho)} = B_{21}(0) \exp(c_1^{(21)} m_\rho^2),$$

from their asymptotic form $6z(1-z)$, as well as the normalization constants for the DA of isovector resonances of spin three.

In analysis of experiments on two pion diffractive production off nucleon (see e.g. [7]) the non-resonant background is described by Söding parametrization [16], which takes into account rescattering of produced pions on final nucleon. Let us note however that in a case of hard ($Q^2 \rightarrow \infty$) diffractive production the final state interaction of pions with residual nucleon is suppressed by powers of $1/Q^2$ relative to the leading twist amplitude. Here we proposed alternative leading-twist parametrization (31) describing the so-called “skewing” of two pion spectrum.

Acknowledgments

I am grateful to D. Diakonov, M. Diehl, L. Frankfurt, K. Goeke, P. Pobylitsa, A. Radyushkin, A. Schäfer, M. Strikman, O. Teryaev and C. Weiss for inspiring discussions. This work has been supported in part by the BMFB (Bonn), Deutsche Forschungsgemeinschaft (Bonn) and by COSY (Jülich).

References

- [1] M.V. Polyakov, hep-ph/9809483.
- [2] J. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. **D56** (1997) 2982.

- [3] A. V. Radyushkin, Phys. Rev. **D56** (1997) 5524.
- [4] X. Ji, J. Phys. **G24** (1998) 1181.
- [5] M. Diehl, T. Gousset, B. Pire and O. Teryaev, Phys. Rev. Lett. **81** (1998) 1782.
- [6] M.V. Polyakov and C. Weiss, Bochum University preprint RUB-TPII-7/98, hep-ph/9806390, Phys. Rev. D in press.
- [7] ZEUS Collaboration, hep-ex/9808020.
- [8] G.P. Lepage and S.J. Brodsky, Phys. Lett. **B 87** (1979) 359; A.V. Efremov and A.V. Radyushkin, Phys. Lett. **B 94** (1980) 245.
- [9] M.A. Shifman and M.I. Vysotsky, Nucl. Phys. **B 186**, 475 (1981).
- [10] K.M. Watson, Phys. Rev. **95** (1955) 228.
- [11] R. Omnes, Nuovo Cim. **8** (1958) 316.
- [12] P. Ball and V. M. Braun, Phys. Rev. **D54** (1996) 2186.
- [13] A.P. Bakulev and S.V. Mikhailov, Preprint JINR-E2-97-419 (1998), hep-ph/9803298.
- [14] Particle Data Group, Phys. Rev. **D54** (1996), 1.
- [15] M.V. Polyakov and C. Weiss, Bochum University preprint, RUB-TP2-01/99.
- [16] P. Söding, Phys. Lett. **B19** (1966) 702.